COMPLEX HYSTERESIS MODELING OF A BROAD CLASS OF HYSTERETIC ACTUATOR NONLINEARITIES

K. Kuhnen, H. Janocha
Laboratory for Process Automation (LPA), Saarland University, Saarbrücken, Germany

Abstract
A big problem in many technical applications is the complex hysteretic characteristic of sensors and actuators, which causes a non-linear and multi-valued mapping between the output variable and the input variable of the transducer. In the last few years at the LPA a general modeling and compensator design method was developed for such complex hysteretic nonlinearities which is based on the so-called modified Prandtl-Ishlinskii approach and which is well suited for real-time applications. The goal of the present paper is to demonstrate experimentally the applicability of this new modeling and compensation method to a broad class of complex hysteretic actuator nonlinearities typically occuring in mechatronic systems with active materials and other actuator principles. With this method the nonlinearity error of the investigated actuator characteristics is reduced at least by a factor of about 15.

Introduction

The proceeding miniaturisation of mechatronic systems requires in addition to the sensor principles, which can be miniaturised, also actuators with a high energy density, to be able to achieve sufficiently big forces in connection with small dimensions. This requirement is largely fulfilled by actuators, which are made of magnetostrictive and piezoelectric materials, shape memory alloys, by electromagnetic actuators and so on. But one of the biggest problems in control is the complex hysteretic characteristic of sensors and actuators, which causes a non-linear and multi-valued mapping between the output variable and the input variable of the transducer. Fig. 1 shows some examples of well-known actuator characteristics. Here s is the displacement, U the electrical voltage and I the current of the corresponding actuator principle. Thereby the non-linear and multi-valued relation between the corresponding input and output variables has a very different characteristic depending on the type of the actuator. One possibility to handle this problem is to compensate the hysteretic actuator characteristic in an open loop control by using an inverse feed-forward controller. In contrast to a closed-loop compensation scheme this solution needs no additional sensor for the acquisition of the actuators output variable. Furthermore thanks to the use of a stable inverse feed-forward controller there is never a risk of instability of the total system. In the last few years at the LPA a general modeling and compensator design method was developed for such complex hysteretic nonlinearities which is based on the so-called modified Prandtl-Ishlinskii approach [4,5]. The goal of the present paper is to demonstrate experimentally the applicability of this new modeling and compensation method to a broad class of complex hysteretic actuator nonlinearities typically occuring in smart material and mechatronic systems.

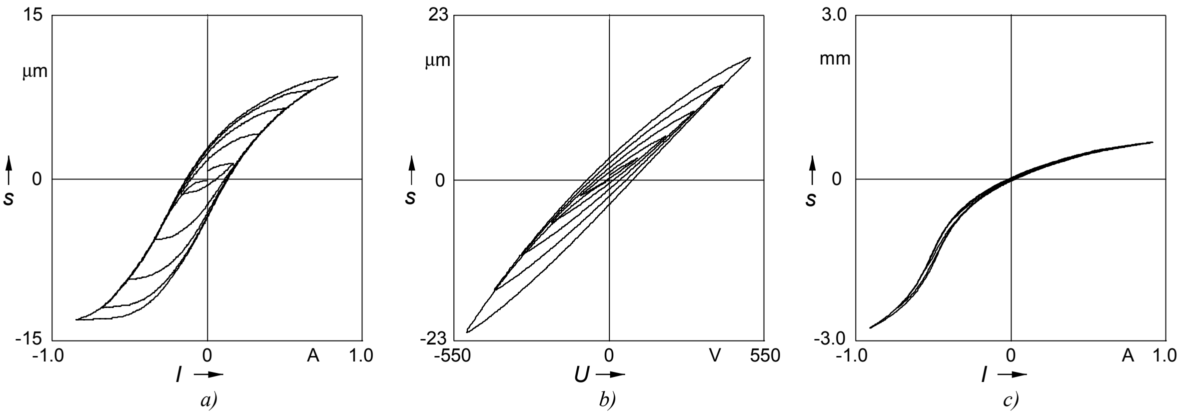

Fig. 1: *Measured actuator characteristics of different transducer principles*
a) Magnetostrictive actuator b) Piezoelectric actuator c) Electromagnetic actuator

Modeling and compensation of complex hysteretic nonlinearities

In the mathematical literature the notation of the hysteretic nonlinearity will be equated with the notation "rate independent memory effect" [1,2,7]. This means that the output signal of a system with hysteresis depends not only on the present value of the input signal but also on the order of their amplitudes, especially their extremum values, but not on their rate in the past. The rate-independent branching transfer characteristic shown in Fig. 1 is a typical sign of a system with hysteretic nonlinearities which can be divided into two classes. In the first class the hysteretic nonlinearity has a local memory structure which means that the present value of the output is only dependent on the present value and one extremum value of the input. But nearly all hysteretic nonlinearities which occur in smart material based sensor and actuator characteristics have a non-local respectively complex memory structure and thus belong to the second class. In this case the present value of the output is not only dependent on the present value of the input, but also on more than one to infinite many extremum values of the input in the past. A possible consequence of this complex memory structure are closed major and minor loops and the intersection of branches in the same direction of the input signal.

In the beginning mainly the well-known Preisach operator was used for the modeling and linearization of complex hysteretic nonlinearities occuring in solid-state actuators with the inverse feed-forward control approach [6]. But the main drawback of the Preisach operator is the fact that in general the compensator of the Preisach hysteresis operator has to be calculated numerically which is not suitable for real-time applications. Recent papers also reference the so-called Prandtl-Ishlinskii hysteresis operator [3]. The elementary hysteretic kernel within the Prandtl-Ishlinskii hysteresis operator is the so-called play operator H_{r_H} which is defined by

$$y(t) = \max\{x(t) - r_H, \min\{x(t) + r_H, y(t_i)\}\} \quad (1)$$

with the initial consistency condition

$$y(t_0) = \max\{x(t_0) - r_H, \min\{x(t_0) + r_H, y_0\}\} \quad (2)$$

for the output signal at initial time t_0 for piecewise monotonous input signals with a monotonicity partition $t_0 \leq t_1 \leq .. \leq t_i \leq t \leq t_{i+1} .. \leq t_N = t_E$ [1]. The play operator depends on the independent initial value $y_0 \in \Re$ of the output and is characterized by its threshold parameter $r_H \in \Re^+_0$. Fig. 2 shows the rate-independent output-input trajectory of this elementary hysteresis operator.

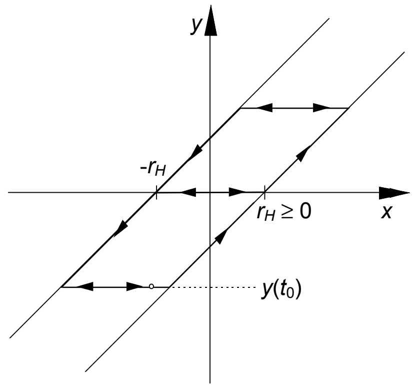

Fig. 2: Rate-independent characteristic of the play operator

The Prandtl-Ishlinskii hysteresis operator is in its threshold-discrete form defined by

$$H[x](t) := \boldsymbol{w}_H^{\,T} \cdot \boldsymbol{H}_{r_H}[x, \boldsymbol{z}_{H0}](t) \quad (3)$$

with the vector of weights $\boldsymbol{w}_H^T = (w_{H0}\ w_{H1}\ ..\ w_{Hn})$, the vector of thresholds $\boldsymbol{r}_H^T = (r_{H0}\ r_{H1}\ ..\ r_{Hn})$ with $0 = r_{H0} < r_{H1} < .. < r_{Hn} < +\infty$, the vector of the initial states $\boldsymbol{z}_{H0}^T = (z_{H00}\ z_{H01}\ ..\ z_{H0n})$ and the vector of the play operators

$$\boldsymbol{H}_{r_H}^{\,T} = (H_{r_{H0}}\ H_{r_{H1}}\ ..\ H_{r_{Hn}}) .$$

The main advantages of this approach are the reduced model complexity of the Prandtl-Ishlinskii operator in comparision with the Preisach operator and the fact that the compensator of an invertible Prandtl-Ishlinskii operator is also of Prandtl-Ishlinskii type, which means

$$H^{-1}[y](t) = \boldsymbol{w}'^{\,T}_H \cdot \boldsymbol{H}_{r'_H}[y, \boldsymbol{z}'_{H0}](t) , \quad (4)$$

and can be calculated analytically by using the weight, theshold and initial state transformation laws $\boldsymbol{r}_H' = \boldsymbol{\Omega}_H(\boldsymbol{r}_H, \boldsymbol{w}_H)$, $\boldsymbol{w}_H' = \boldsymbol{\Xi}_H(\boldsymbol{w}_H)$ and $\boldsymbol{z}_{H0}' = \boldsymbol{\Psi}_H(\boldsymbol{z}_{H0}, \boldsymbol{w}_H)$ which are discussed in detail in [3,5]. This allows an efficient implementation of the compensator for real-time applications. On the other side the closed loops of the Prandtl-Ishlinskii hysteresis operator have an odd symmetry to the center point of the corresponding loop. This is a property of the underlying play operator and persists under linear superposition. This main drawback of the Prandtl-Ishlinskii hysteresis modeling approach is often too restrictive for real complex hysteretic actuator nonlinearities, see Fig. 1, and therefore reduce its applicability in practice.

An intuitive idea to overcome these restrictions is to combine in series the hysteresis operator and a continuous, non convex and non symmetrical memory-free nonlinearity. This leads to the so-called modified Prandtl-Ishlinskii hysteresis modeling approach which uses a special modeling technique for memory-free nonlinearities. It bases on the weighted superposition of so-called one-sided dead-zone operators S_{r_S} which are defined by the function

$$S(x(t), r_S) = \begin{cases} \max\{x(t) - r_S, 0\} & ; \quad r_S > 0 \\ x(t) & ; \quad r_S = 0 \\ \min\{x(t) - r_S, 0\} & ; \quad r_S < 0 \end{cases} \quad (5)$$

between the present values of the corresponding output and input signal. This elementary superposition operator is also fully characterized by a threshold parameter $r_S \in \Re$. Fig. 3 shows the rate-independent output-input trajectory of this elementary superposition operator for different threshold values.

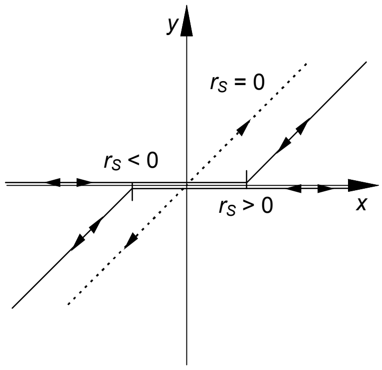

Fig. 3: *Rate-independent characteristic of the one-sided dead-zone operator*

The complex superposition operator for the approximation of more general continuous memory-free nonlinearities is in its threshold-discrete form given by

$$S[x](t) := \boldsymbol{w}_S^{\,T} \cdot \boldsymbol{S}_{r_S}[x](t) \quad (6)$$

with the vector of weights $\boldsymbol{w}_S^T = (w_{S-l} \,..\, w_{S0} \,..\, w_{Sl})$, the vector of thresholds $\boldsymbol{r}_S^T = (r_{S-l} \,..\, r_{S0} \,..\, r_{Sl})$ with $-\infty < r_{S-l} < .. < r_{S0} = 0 < .. < r_{Sl} < +\infty$ and the vector of the one-sided dead-zone operators

$$\boldsymbol{S}_{r_S}^{\,T} = (S_{r_{S-l}} .. S_{r_{S0}} .. S_{r_{Sl}}) .$$

The compensator of a memory-free nonlinearity described by an invertible Prandtl-Ishlinskii superposition operator is also of Prandtl-Ishlinskii type, which means

$$S^{-1}[y](t) = \boldsymbol{w}_S'^{\,T} \cdot \boldsymbol{S}_{r_S'}[y](t) , \quad (7)$$

and can be calculated analytically in the same way as the inverse Prandtl-Ishlinskii hysteresis operator by using the corresponding weight and theshold transformation laws $\boldsymbol{r}_S' = \boldsymbol{\Omega}_S(\boldsymbol{r}_S, \boldsymbol{w}_S)$ and $\boldsymbol{w}_S' = \boldsymbol{\Xi}_S(\boldsymbol{w}_S)$ which are discussed in detail in [4,5]. Because of this properties, which are very similar to the properties of H, the quantity S is called the Prandtl-Ishlinskii superposition operator.
With the Prandtl-Ishlinskii hysteresis and superposition operators H and S the modified Prandtl-Ishlinskii hysteresis operator is defined by

$$\Gamma[x](t) := \boldsymbol{w}_S^{\,T} \cdot \boldsymbol{S}_{r_S}[\boldsymbol{w}_H^{\,T} \cdot \boldsymbol{H}_{r_H}[x, \boldsymbol{z}_{H0}]](t) \quad (8)$$

and the inverse modified Prandtl-Ishlinskii hysteresis operator can be easily obtained by the inversion of the Prandtl-Ishlinskii hysteresis operator and the Prandtl-Ishlinskii superposition operator and an exchange of their order. From this follows

$$\Gamma^{-1}[y](t) = \boldsymbol{w}_H'^{\,T} \cdot \boldsymbol{H}_{r_H'}[\boldsymbol{w}_S'^{\,T} \cdot \boldsymbol{S}_{r_S'}[y], \boldsymbol{z}_{H0}'](t) . \quad (9)$$

Thus the modified Prandtl-Ishlinskii hysteresis modeling approach permits the consistent modeling and compensation of invertable complex hysteretic nonlinearities with asymmetrical loops.

Experimental results and discussion

In this section the performance of modified Prandtl-Ishlinskii hysteresis modeling approach will now be demonstrated by means of the different complex hysteretic actuator nonlinearities shown in Fig. 1. The hysteretic actuator characteristic of the piezoelectric transducer in Fig. 1b for example shows a strong hysteretic branching with a weak deviation from the odd symmetry to the origin. In this case a modified Prandtl-Ishlinskii hysteresis operator with 8 play operators and 6 one-sided dead-zone operators is suffucient for a good correspondence with the measured hysteretic characteristic. However, the hysteretic actuator characteristic of the electromagnetic transducer in Fig. 1c shows a weak hysteretic branching with a strong deviation from the odd symmetry to the origin. In this case a modified Prandtl-Ishlinskii hysteresis operator with 6 play operators and 24 one-sided dead-zone operators is used to obtain a good correspondence with the measured nonlinear characteristic. Finally, the hysteretic actuator characteristic of the magnetostrictive transducer in Fig. 1a shows a strong hysteretic branching as well as a strong deviation from the odd symmetry to the origin. In this case a modified Prandtl-Ishlinskii hysteresis operator with 14 play operators and 14 one-sided dead-zone operators is used to obtain a good correspondence with the measured nonlinear characteristic.

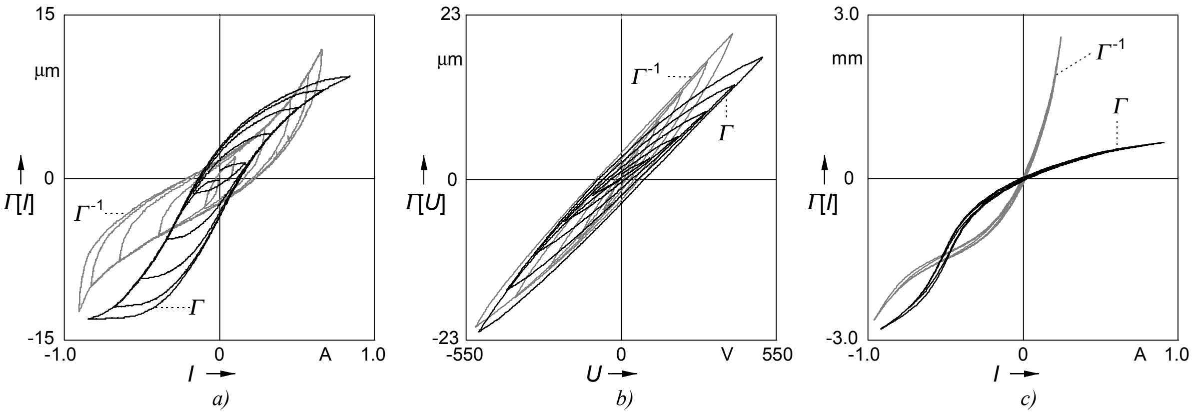

Fig. 4: *Modified Prandtl-Ishlinskii operators and the corresponding inverses for the different actuator characteristics shown in Fig. 1 a) Magnetostrictive actuator b) Piezoelectric actuator c) Electromagnetic actuator*

Fig. 4a-c show the branching of the modified Prandtl-Ishlinskii hysteresis operators as a black line and the corresponding exact inverse operators as a gray line which are identified by means of the corresponding measured characteristics of the magnetostrictive, piezoelectric and electromagnetic actuators shown in Fig. 1a-c. Table 1 shows the nonlinearity error defined by

$$e = \frac{\max\limits_{t_0 \le t \le t_e} \{ |\Gamma[x](t) - y(t)| \}}{\max\limits_{t_0 \le t \le t_e} \{ |\Gamma[x](t)| \}} \qquad (10)$$

in comparison with the best linear approximation ($n = 0$, $l = 0$) whereby $x \equiv I$ and $y \equiv s$ for the magnetostrictive and the electromagnetic actuator, $x \equiv U$ and $y \equiv s$ for the piezoelectric actuator.

Actuator type	*n*	*l*	*e*
Magnetostrictive actuator	0	0	48.7 %
	14	7	2.6 %
Piezoelectric actuator	0	0	17.3 %
	8	3	1.2 %
Electromagnetic actuator	0	0	63.1 %
	6	12	1.6 %

Table 1: *Nonlinearity errors*

In comparison to the best linear approximation the use of the modified Prandtl-Ishlinskii hysteresis modeling approach reduces the nonlinearity error strongly to factors of about 20 for the magnetostrictive actuator, about 15 for the piezoelectric actuator and about 40 for the electromagnetic actuator.

Summary and prospects

This paper has demonstrated the applicability of the modified Prandtl-Ishlinskii hysteresis modeling approach to totally different complex hysteretic actuator nonlinearities occuring in magnetostrictive, piezoelectric and electromagnetic transducers. In all these cases the nonlinearity error was reduced at least by a factor of about 15. In the meantime the method is available as a commercial software product [8]. A real-time effective extension of the modified Prandtl-Ishlinskii approach to complex creep dynamics occuring especially in the characteristic of piezoelectric actuators is described in [5]. In future works the application of the method also to shape-memory actuators is planned.

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