

Linear coefficient identification in 2D

Linear error model:
$$e_p^n(\mathbf{y}, r/r_0, \mathbf{p}) = \mathbf{1}_{qau}^n(\mathbf{y})^T \cdot \mathbf{p} - (r/r_0^n - 1)$$

Quality function:
$$V_p(\mathbf{p}) = \frac{1}{2} \sum_{n=1}^N e_p^n(\mathbf{y}, r/r_0, \mathbf{p})^2 = \frac{1}{2} \mathbf{p}^T \cdot \mathbf{A} \cdot \mathbf{p} + \mathbf{b}^T \cdot \mathbf{p} + c$$

Data base:
$$\mathbf{A} = \sum_{n=1}^N \mathbf{1}_{qau}^n(\mathbf{y}) \cdot \mathbf{1}_{qau}^n(\mathbf{y})^T, \quad \mathbf{b} = -\sum_{n=1}^N \mathbf{1}_{qau}^n(\mathbf{y}) \cdot (r/r_0^n - 1), \quad c = \frac{1}{2} \sum_{n=1}^N (r/r_0^n - 1)^2$$

Quality function gradient:
$$\frac{\partial}{\partial \mathbf{p}} V_p(\mathbf{p}) = \mathbf{A} \cdot \mathbf{p} + \mathbf{b}$$

Positivity constraints:
$$K_p = \left\{ \mathbf{p} \mid \mathbf{U}_p \cdot \mathbf{p} - \mathbf{u}_p \geq \mathbf{o} \right\},$$

$$\underbrace{\begin{pmatrix} \mathbf{U}_{11} & \cdots & \mathbf{O} \\ \vdots & \ddots & \vdots \\ \mathbf{O} & \cdots & \mathbf{U}_{JK} \end{pmatrix}}_{\mathbf{U}_p} \cdot \underbrace{\begin{pmatrix} \mathbf{p}_{11} \\ \vdots \\ \mathbf{p}_{JK} \end{pmatrix}}_{\mathbf{p}} - \underbrace{\begin{pmatrix} \mathbf{u}_{11} \\ \vdots \\ \mathbf{u}_{JK} \end{pmatrix}}_{\mathbf{u}_p} \geq \underbrace{\begin{pmatrix} \mathbf{o} \\ \vdots \\ \mathbf{o} \end{pmatrix}}_{\mathbf{o}}, \quad \underbrace{\begin{pmatrix} 1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 1 & \cdots & 1 \end{pmatrix}}_{\mathbf{U}_{jk}} \cdot \underbrace{\begin{pmatrix} \mathbf{p}_1 \\ \vdots \\ \mathbf{p}_l \end{pmatrix}}_{\mathbf{p}_{jk}} - \underbrace{\begin{pmatrix} \varepsilon - \frac{1}{J \cdot K} \\ \vdots \\ \varepsilon - \frac{1}{J \cdot K} \end{pmatrix}}_{\mathbf{u}_{jk}} \geq \underbrace{\begin{pmatrix} 0 \\ \vdots \\ 0 \end{pmatrix}}_{\mathbf{o}}$$

Identification problem:
$$\mathbf{p}^\# = \underset{\mathbf{p} \in K_p}{\operatorname{argmin}} \{V_p(\mathbf{p})\}$$

Threshold, angle and stretch constraints in 2D

Threshold constraints:

$$\underbrace{\begin{pmatrix} +1 & 0 & \dots & 0 & 0 \\ -1 & +1 & \dots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \dots & -1 & +1 \\ 0 & 0 & \dots & 0 & -1 \end{pmatrix}}_{\mathbf{u}_q} \cdot \underbrace{\begin{pmatrix} q_1 \\ q_2 \\ \vdots \\ q_{l-1} \\ q_l \end{pmatrix}}_{\mathbf{q}} - \underbrace{\begin{pmatrix} \varepsilon \\ \varepsilon \\ \vdots \\ \varepsilon \\ \varepsilon - y_{\max} \end{pmatrix}}_{\mathbf{u}_q} \geq \underbrace{\begin{pmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 0 \end{pmatrix}}_{\mathbf{o}}$$

Angle constraints:

$$\underbrace{\begin{pmatrix} +1 & 0 & \dots & 0 & 0 \\ -1 & +1 & \dots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \dots & -1 & +1 \\ 0 & 0 & \dots & 0 & -1 \end{pmatrix}}_{\mathbf{u}_\alpha} \cdot \underbrace{\begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \vdots \\ \alpha_{j-1} \\ \alpha_j \end{pmatrix}}_{\boldsymbol{\alpha}} - \underbrace{\begin{pmatrix} 0 \\ \varepsilon \\ \vdots \\ \varepsilon \\ -\pi/2 \end{pmatrix}}_{\mathbf{u}_\alpha} \geq \underbrace{\begin{pmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 0 \end{pmatrix}}_{\mathbf{o}}$$

Stretch constraints:

$$\underbrace{\begin{pmatrix} +1 & 0 & \dots & 0 & 0 \\ -1 & +1 & \dots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \dots & -1 & +1 \\ 0 & 0 & \dots & 0 & -1 \end{pmatrix}}_{\mathbf{u}_v} \cdot \underbrace{\begin{pmatrix} v_1 \\ v_2 \\ \vdots \\ v_{k-1} \\ v_k \end{pmatrix}}_{\mathbf{v}} - \underbrace{\begin{pmatrix} +1 \\ \varepsilon \\ \vdots \\ \varepsilon \\ -10 \end{pmatrix}}_{\mathbf{u}_v} \geq \underbrace{\begin{pmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 0 \end{pmatrix}}_{\mathbf{o}}$$