Antisymmetric polynomial for C^4-regularized piecewise quadratic potential reconstruction

C^4-regularization polynom

$$\int S^{\kappa\varepsilon}\,dz = P^{\kappa\varepsilon}(z) = \begin{cases} m_{off} & , \quad z \le -\kappa \\ m_{-1} + m_0 z + \frac{1}{2} m_1 z^2 + \frac{1}{3} m_2 z^3 + \frac{1}{4} m_3 z^4 & , \quad -\kappa < z \le 0 \\ p_{-1} + p_0 z + \frac{1}{2} p_1 z^2 + \frac{1}{3} p_2 z^3 + \frac{1}{4} p_3 z^4 & , \quad 0 < z \le +\varepsilon \\ p_{off} + \upsilon z + \frac{1}{2} z^2 & , \quad +\varepsilon < z \end{cases}$$

$$m_{off} = 0\,, \quad m_{-1} = +m_0\kappa - \frac{1}{2} m_1 \kappa^2 + \frac{1}{3} m_2 \kappa^3 - \frac{1}{4} m_3 \kappa^4$$

$$p_{-1} = m_{-1}, \quad p_{off} = -\upsilon\varepsilon - \frac{1}{2}\varepsilon^2 + p_{-1} + p_0\varepsilon + \frac{1}{2} p_1 \varepsilon^2 + \frac{1}{3} p_2 \varepsilon^3 + \frac{1}{4} p_3 \varepsilon^4$$

Threshold-dependent quantities

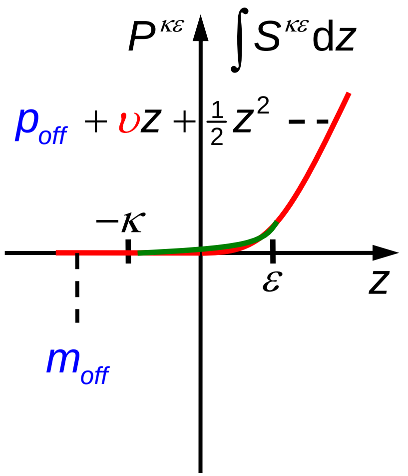
$$z = x - s_i$$

$$\kappa = r \cdot (s_i - s_{i-1})$$

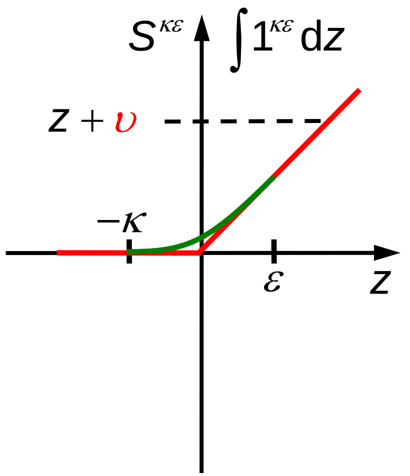
$$\varepsilon = r \cdot (s_{i+1} - s_i)$$

$$0 \le r \le 1$$

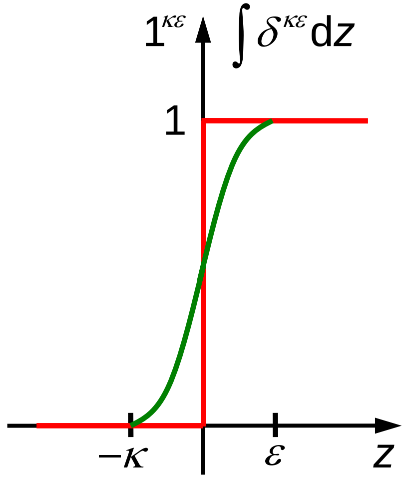
Potential regularization



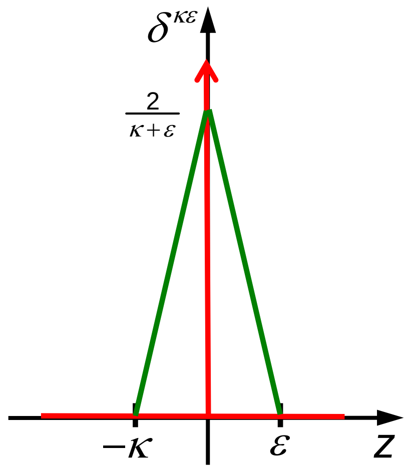
Ramp regularization



Step regularization



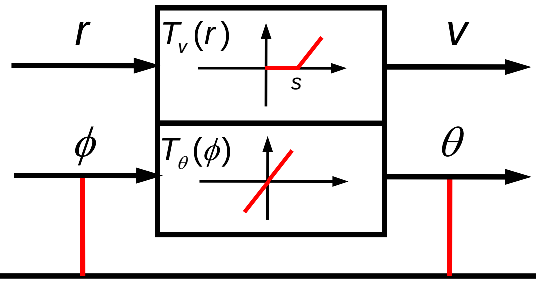
Impulse regularization



Kuhnen, K.: "Construction of complex anisotropic potentials with piecewise quadratic basis functions", Leonberg, Germany, 2018.

Potential, Gradient and Hessian regularization of anisotropic piecewise quadratic potentials in 2D

Non-smooth transformation:



Gradient regularization:

Potential regularization:

$$\nabla p_{s\varphi\eta}(r,\phi) = \underbrace{\begin{pmatrix} 1_s(r) & 0 \\ 0 & 1 \end{pmatrix}}_{\boldsymbol{J}^T(r,\phi)} \underbrace{\begin{pmatrix} T_v(r) \\ 0 \end{pmatrix}}_{\nabla p(v,\theta)} = \underbrace{\begin{pmatrix} S_s(r) \\ 0 \end{pmatrix}}_{\nabla p(r,\phi)} \to \nabla p^{\kappa\varepsilon}_{s\varphi\eta}(r,\phi) = \begin{pmatrix} S_s^{\kappa\varepsilon}(r) \\ 0 \end{pmatrix} \leftarrow P_s^{\kappa\varepsilon}(r) = p^{\kappa\varepsilon}_{s\varphi\eta}(r,\phi)$$

Hessian regularization:

$$\nabla^2 p_{s\varphi\eta}(r,\phi) = \boldsymbol{J}^T(r,\phi)\nabla^2 p(v,\theta)\boldsymbol{J}(r,\phi) + \frac{\partial p(v,\theta)}{\partial v}\nabla^2 T_v(r) + \frac{\partial p(v,\theta)}{\partial \theta}\nabla^2 T_\theta(\phi)$$

$$= \underbrace{\begin{pmatrix} 1_s(r) & 0 \\ 0 & 1 \end{pmatrix}}_{\boldsymbol{J}^T(r,\phi)} \underbrace{\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}}_{\nabla^2 p(v,\theta)} \underbrace{\begin{pmatrix} 1_s(r) & 0 \\ 0 & 1 \end{pmatrix}}_{\boldsymbol{J}(r,\phi)} + \underbrace{S_s(r)}_{\frac{\partial p(v,\theta)}{\partial v}} \underbrace{\begin{pmatrix} \delta_s(r) & 0 \\ 0 & 0 \end{pmatrix}}_{\nabla^2 T_v(r)} + \underbrace{0}_{\frac{\partial p(v,\theta)}{\partial \theta}} \underbrace{\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}}_{\nabla^2 T_\theta(\phi)} = \begin{pmatrix} 1_s(r) & 0 \\ 0 & 0 \end{pmatrix}$$

$$\downarrow$$

$$\nabla^2 p^{\kappa\varepsilon}_{s\varphi\eta}(r,\phi) = \begin{pmatrix} 1_s^{\kappa\varepsilon}(r) & 0 \\ 0 & 0 \end{pmatrix}$$

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