INVERSE FEEDFORWARD CONTROLLER FOR COMPLEX HYSTERETIC NONLINEARITIES IN SMART-MATERIAL SYSTEMS

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Abstract

Undesired complex hysteretic nonlinearities are present to a varying degree in virtually all smart material-based sensors and actuators provided that they are driven with sufficiently high amplitudes. In motion and active vibration control applications, for example, these nonlinearities can excite unwanted dynamics which leads in the best case to reduced system performance and in the worst case to unstable system operation. This necessitates the development of purely phenomenological models which characterize these nonlinearities in a way which is sufficiently accurate, amenable to a compensator design for actuator linearization and efficient enough for use in real-time applications. To fulfil these demanding requirements the present paper describes a new compensator design method for invertible complex hysteretic nonlinearities which is based on the so-called Prandtl-Ishlinskii hysteresis operator. The parameter identification of this model can be formulated as a quadratic optimization problem which produces the best L_2^2-norm approximation for the measured output-input data of the real hysteretic nonlinearity. Special linear inequality constraints for the parameters guarantee the unique solvability of the identification problem and the invertability of the identified model. This leads to a robustness of the identification procedure against unknown measurement errors, unknown model errors and unknown model orders. The corresponding compensator can be directly calculated and thus efficiently implemented from the model by analytical transformation laws. Finally the compensator design method is used to generate an inverse feedforward controller for the linearization of a magnetostrictive actuator. In comparision to the conventionally controlled magnetostrictive actuator the nonlinearity error of the inverse controlled magnetostrictive actuator is lowered from about 30 % to about 3 %.

Key Words

Hysteresis, Nonlinear Systems, Modeling, Identification, Compensation

1. Introduction

Complex memory-free nonlinearities or in generalization complex hysteretic nonlinearities are present to a varying degree in virtually all smart material-based sensors and actuators provided that they are driven with sufficiently high amplitudes. Well-known complex hysteretic nonlinearities are the magnetic induction - magnetic field relation of ferromagnetic materials, the electrical polarization - electrical field relation of ferroelectric materials and the stress - strain relation of elasto-plastic materials. The most familiar examples for complex hysteretic nonlinearities in smart material systems are piezoelectric, magnetostrictive and shape memory-alloy based actuators and sensors [1]. In many applications, these nonlinearities can be limited through the choice of proper materials and operating regimes so that linear sensor and actuator characteristics can be assumed. In the consequence of more stringent performance requirements a large number of actuators are currently operated in regimes in which hysteretic nonlinearities are unavoidable. In motion and active vibration control applications, for example, these nonlinearities can excite unwanted dynamics which leads in the best case to reduced closed-loop system performance and in the worst case to unstable closed-loop system operation, see fig. 1.

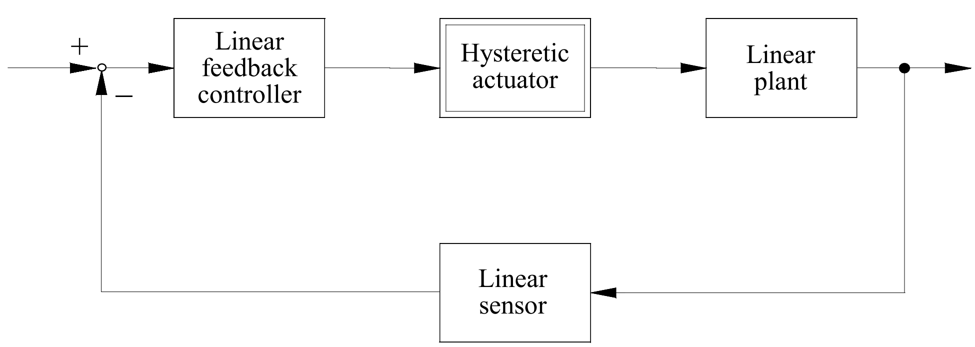

figure 1. Closed-loop control application with a hysteretic actuator nonlinearity

This necessitates the development of purely phenomenological models which characterize these nonlinearities in a way which is sufficiently accurate, amenable to a compensator design for actuator linearization and efficient enough for use in real-time applications.

Models of hysteretic nonlinearities have evolved from two different branches of physics: ferromagnetism and plasticity theory. The roots of both branches go back to the end of the 19th century. But only at the beginning of the 1970s was a mathematical formalism for a systematic consideration of hysteretic nonlinearities developed [2]. The core of this theory is formed by so-called hysteresis operators which describe hysteretic transducers as a mapping between function spaces. But it is only since the beginning of the 1990s that engineers employ this theory on a larger scale to develop modern strategies for the linearization of hysteretic nonlinearities with an inverse feedforward controller shown in fig 2.

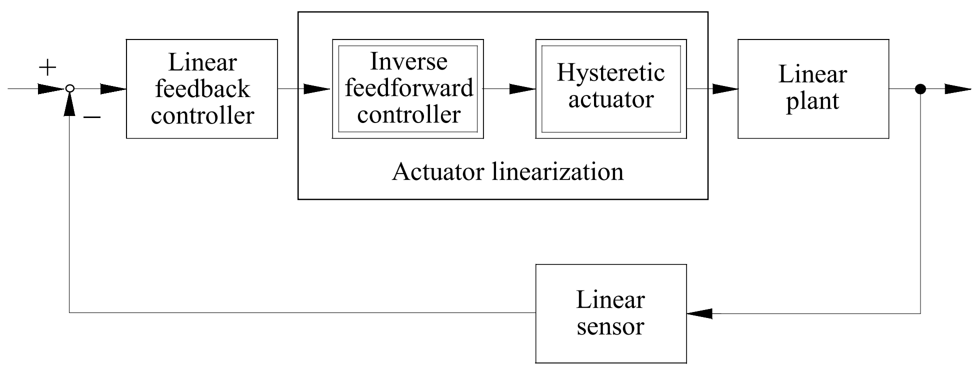

figure 2. Linearization of a hysteretic actuator by an inverse feedforward control strategy

This type of controller is based on the compensator W^{-1} of the hysteretic nonlinearity W which is defined as the operator fulfilling

$$W[W^{-1}[x]](t) = I[x](t)\,. \tag{1}$$

In (1) I is the identity operator which describes the idealized transfer characteristic

$$I[x](t) = x(t)\,. \tag{2}$$

There are many nonlinear control systems with hysteresis operating in practice successfully, many of which have been designed using techniques as the describing functions. But the main drawback of these solutions is the limitation to harmonic input signals [3]. In contrast to this, the linearization of hysteretic nonlinearities W with an inverse feedforward controller based on its compensator W^{-1} is not restricted to a special input function and thus much more general in its applicability. The inverse feedforward control approach for the linearization of hysteretic nonlinearities can be divided into two classes. In the first class the underlying hysteretic nonlinearity has a local respectively non-complex memory structure which means that the present value of the output is only dependent on the present value and the last extremum value of the input. But nearly all hysteretic nonlinearities which occur in smart material based sensor and actuator characteristics have a non-local respectively complex memory structure and thus belong to the second class. In this case the present value of the output is not only dependent on the present value of the input, but also on more than one extremum value of the input in the past.

In the beginning mainly the well-known Preisach operator was used for the modeling and linearization of complex hysteretic nonlinearities occuring in solid-state actuators with the inverse feedforward control approach [4,5]. But the main drawbacks of the Preisach operator are the strong sensitivity of the identification procedure against output-input data and unknown model errors and the fact that in general the compensator of the Preisach operator has to be calculated numerically. Recent papers also reference the so-called Prandtl-Ishlinskii operator [6,7,8] which belongs to an important subclass of the Preisach operator [9]. The main advantages of this approach are the reduced model complexity of the Prandtl-Ishlinskii operator in comparision with the Preisach operator and the fact that the compensator of an invertible Prandtl-Ishlinskii operator can be calculated analytically. This allows an efficient implementation of the compensator for real-time applications.

Developing a consistent phenomenological design concept for a compensator W^{-1} of an invertible complex hysteretic nonlinearity W which is sufficiently flexible in its modeling capabilities and moreover well suited for real-time applications is in general not a simple task because it covers the following coupled design steps: modeling the real hysteretic nonlinearity, identification of the model parameters to adapt the model to the real hysteretic nonlinearity and inversion of the model to obtain the desired compensator. Especially the mathematical complexity of the identification and inversion problem depends on the phenomenological modeling method (for example Preisach or Prandtl-Ishlinskii modeling) and influences strongly the practical use of the design concept. Another difficulty of the identification problem follows from the strong sensitivity of the model parameters to unknown measurement errors of the output-input data, unknown model errors and unknown model orders. Due to these effects a parameter identification can result in the best case to a poor model accuracy or in the worst case to a locally non-invertible model, and as a consequence the whole compensator design fails. Therefore the robustness against these effects is an inherent requirement for a consistent phenomenological compensator design method.

To overcome these difficulties the present paper describes a new compensator design concept for complex hysteretic nonlinearities based on the Prandtl-Ishlinskii modeling approach which is robust in the sense mentioned above. The robustness of the new compensator design method is reached by the consideration of linear inequality constraints for the free model parameters which guarantee a search for the best L_2^2-norm approximation of the measured output-input data only in those parameter ranges where the identified model is invertible.

2 Hysteresis definition and observable characteristics

What is a scalar hysteretic nonlinearity? To answer this question, let us consider a system *W*, i.e. a mapping

$$y(t) = W[x](t) \tag{3}$$

for all $t \in [t_0, t_E]$ with a scalar input signal x and a scalar output signal y, each of which is continuously dependent upon the time t. The transformation is causal meaning that for a given set of starting conditions

$$x_1(\tau) = x_2(\tau) \quad \text{für} \quad \tau \in [t_0, t] \;\Rightarrow\; W[x_1](t) = W[x_2](t)\,. \tag{4}$$

The time-dependent y-x trajectory of the system can be simply graphically represented by assigning a point in the y-x plane to each $(x(t),y(t))$ pair, see fig.3. Let us assume that the input signal x increases starting from small values and going through the values x_1 and x_2 and beyond. Then the pair $(x(t),y(t))$, and consequently the time-dependent y-x trajectory, follows a path P_1 in the y-x plane. If the input signal changes direction falling from large values through x_2 on to x_1 and below, then the pair $(x(t),y(t))$, and consequently the time-dependent y-x trajectory, follows path P_2. If the input parameter changes direction within the interval $x_1 < x(t) < x_2$, then the pair $(x(t),y(t))$, and consequently the time-dependent y-x trajectory follows path P_3 into the so-called hysteretic region Ω, which is bounded by the major loop described by paths P_1 and P_2.

The major loop need not be closed. The formation of closed hysteresis loops is often observed in practice but is not demanded by the strictly mathematical definition of hysteretic nonlinearity [9,10,11]. An observable, inherent feature of hysteretic nonlinearity is, however, the branching of the time-dependent y-x trajectory upon reversing the direction of the input signal. This branching results from the memory effect inherent of hysteretic nonlinearity. As a result of this effect, the momentary value of the output signal depends not only on the momentary value of the input signal but also on previous input signal values as well as the initial values stored in "memory". This means that different input signal histories and different initial states will lead to different paths in the *y*-*x* plane.

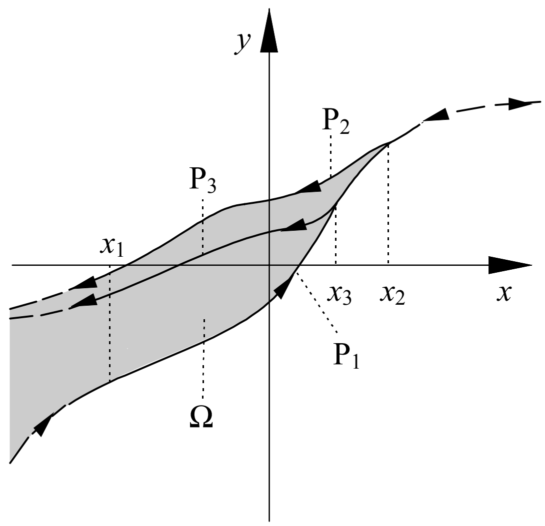

figure 3. Features of hysteretic behaviour

The branching characteristic in the *y*-*x* trajectory is in itself insufficient to indicate hysteretic nonlinearity since essentially all causal transfer functions with memory exhibit branching behaviour. The decisive characteristic differentiating hysteretic nonlinearities from all other systems with memory is its independence of the speed of change of the input signal (rate-independence property). This means that the branching behaviour in the *y*-*x* plane does not depend on the rate of change of the input signal but only on the order of the input signal amplitudes. Mathematically, the rate independence of the transfer function can be expressed by the fundamental expression of the causal operator W

$$W[x \circ \eta](t) = W[x](\eta(t)) \tag{5}$$

for all $t \in [t_0,t_E]$, whereby the properties $\eta(t_0) = t_0$ and $\eta(t_E) = t_E$ must apply in order to fulfil the continuous and monotonous time transformation $\eta : [t_0,t_E] \to [t_0,t_E]$ [9]. This results in

Definition 1: (scalar hysteretic nonlinearity)

A scalar hysteretic nonlinearity W, or a scalar hysteresis operator W, is a mapping that uniquely assigns a scalar time-dependent input signal x to a scalar time-dependent output signal y and that exhibits the properties of causality (4) and rate independence (5).

The set of all scalar hysteretic nonlinearities can be divided into two different classes, namely the class of hysteretic nonlinearities with local memory and the class of hysteretic nonlinearities with non-local memory.

Hysteretic nonlinearities with local memory are characterized by the fact that the value pairs $(x_0 = x(t_0), y_0 = y(t_0)) \in \Omega$ and the values of the input signal $x(t)$ for $t > t_0$ uniquely determine the time dependence of the output signal $y(t)$ for $t > t_0$. This means that in hysteretic nonlinearities with local memory the influence of past input signal amplitudes on the future time dependence of the output signal is taken into consideration in the momentary value of the pair $(x(t),y(t)) \in \Omega$. The state of hysteretic nonlinear systems with local memory is therefore given in the pair $(x(t),y(t)) \in \Omega$ and can be represented graphically in the hysteretic region Ω by a point. Therefore, hysteretic nonlinearities with local memory do not exhibit a real internal state and can therefore be represented mathematically in the form

$$y(t) = W[x, x_0, y_0](t) \tag{6}$$

with the initial conditions $(x_0,y_0) \in \Omega$ for all $t \in [t_0,\mathrm{t_E}]$. The initial system state (x_0,y_0) must lie in the hysteretic region Ω. Since the input signal is independent, the assumption $x_0 = x(t_0)$ is always fulfilled. This means that the hysteretic nonlinearity with local memory can alternatively be characterized by the fact that the initial value of the output parameter $(y_0 = y(t_0)) \in \Sigma(x_0) = (\{x_0\} \times \Re) \cap \Omega$ and the values of the input signal $x(t)$ for $t \geq t_0$ uniquely determine the time dependence of the output signal $y(t)$ for $t > t_0$. In this way, the influence of past input signal amplitudes on the future time dependence of the output signal is included in the momentary value of the output value $y(t) \in \Sigma(x(t)) = (\{x(t)\} \times \Re) \cap \Omega$. In this case, (6) can also be represented by

$$y(t) = W[x, y_0](t) \tag{7}$$

with the initial conditions $y_0 \in \Sigma(x_0)$ for all $t \in [t_0, t_E]$. $\Sigma(x_0) = (\{x_0\} \times \Re) \cap \Omega$ describes the interval illustrated in fig. 4, which can be either open, half open or closed depending on the formation of the hysteretic region. Since the value of the output signal $y(t_0) = W[x, y_0](t_0)$ at time t_0 according to (7) is determined by the initial value of the input signal $x(t_0) = x_0$ and the independent initial value of the output signal y_0, then $y(t_0) = y_0 \in \Sigma(x_0)$ and therefore $(x_0 = x(t_0), y_0 = y(t_0)) \in \Omega$ cannot be required up front. Should $y_0 \notin \Sigma(x_0)$ and $(x_0, y_0) \notin \Omega$ then the equation upon which operators (6) and (7) are based must be augmented by a consistency condition, which implements a projection of $y(t_0)$ into the interval $\Sigma(x_0)$, see fig. 4.

Hysteretic nonlinearities with non-local memory are characterized by the fact that not only the initial value of the output signal $y_0 = y(t_0) \in \Sigma(x_0)$ and the values of the input signal $x(t)$ for $t \geq t_0$ are required to uniquely determine the time dependence of the output signal $y(t)$ for $t > t_0$, but that also the values of the input signal for times $t < t_0$ influence the output signal $y(t)$ for $t > t_0$. This influence is taken into consideration in hysteretic nonlinearities with non-local memory by the initial value z_0 of a real internal state, which, depending on the power of the memory, can be of a higher dimension or even of infinite dimension.

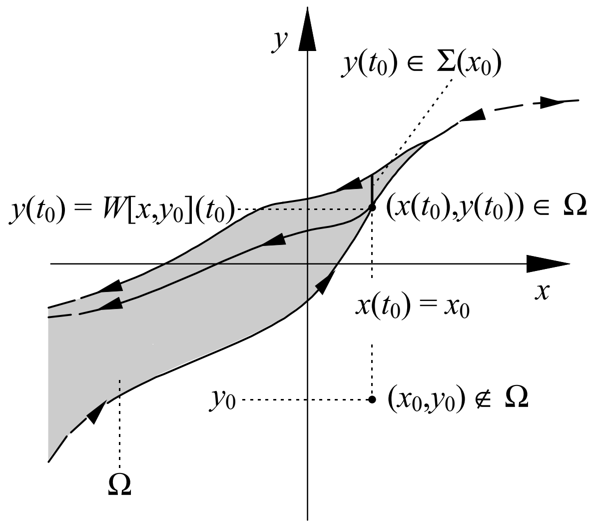

figure 4. Consistency condition for initial values

In the case of a finite dimension the memory is described by a state vector, in the infinite dimension case by a state function. In the latter case, one speaks of hysteretic nonlinearities with global memory. Hysteretic nonlinearities with non-local memory can be described mathematically by the output-input mapping

$$y(t) = W[x, z_0](t) \tag{8}$$

with the initial condition $z_0 \in \Sigma(x_0)$ for all $t \in [t_0,t_E]$. Here, the applicable region of the hysteretic state is denoted by a higher dimensional or infinitely dimensional hysteretic region Σ.

Additionally to the rate-independent branching of the output-input trajectory nearly all smart material based hysteretic nonlinearities show further important characteristics. At the beginning of the 20th century Madelung investigated experimentally the branchings and loopings of ferromagnetic hysteresis and stated the following three rules from his observations [8], see fig. 5.

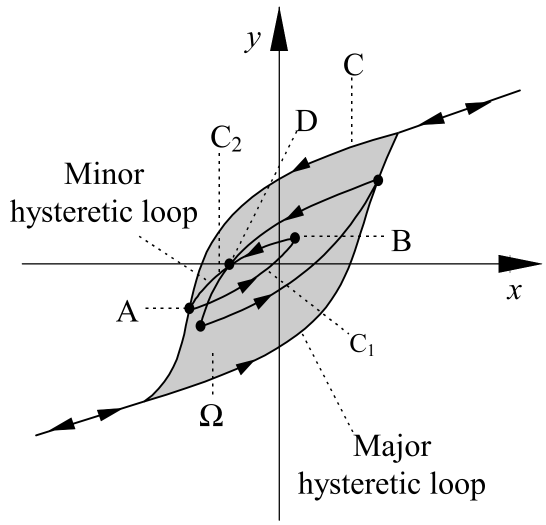

figure 5. Complex hysteretic nonlinearity

1. Any curve C_1 emanating from a turning point A of the output-input trajectory is uniquely determined by the coordinates of A.
2. If any point B on the curve C_1 becomes a new turning point, then the curve C_2 originating at B leads back to the point A.
3. If the curve C_2 is continued beyond the point A, then it coincides with the continuation of the curve C which led to the point A before the C_1-C_2 cycle was traversed.

In addition to these three Madelung rules a fourth important observation can be made for ferromagnetic, ferroelectric, elasto-plastic materials and actuator and sensor characteristics of smart materials.

4. More than one curve can be pass though a non-turning point D within the hysteretic region Ω, see fig. 5. The branch which was traversed is uniquely determined by the relevant past history of the input signal.

It is this so-called crossing-loop property of real hysteretic nonlinearities in which the complex or non-local ones differ from the non-complex or local ones. This property can be understood as a generalization of the first Madelung rule to complex hysteretic nonlinearities.

3 Hysteresis modeling, compensation and identification

Because of its phenomenological character the concept of hysteresis operators allows a powerful modeling of complex hysteretic nonlinearities without taking into account the underlying physics [2]. The basic idea consists of the modeling of the real complex hysteretic nonlinearities by the weighted superposition of many so-called elementary hysteresis operators. Elementary hysteresis operators are non-complex hysteretic nonlinearities with a simple mathematical structure which are characterized by one or more parameters. One of the most familiar and most important elementary hysteretic mapping between the input signal x and the output signal y is the so called play or backlash operator

$$y(t) = H_r[x, y_0](t) \quad \forall \quad r \in \Re_0^+ \quad ; \quad y_0 \in \Re \quad ; \quad t \in [t_0, t_E] \tag{9}$$

which is often used to model mechanical play in gears with one degree of freedom. It is normally defined by the recursive equation

$$y(t) = \max\{x(t) - r, \min\{x(t) + r, y(t_i)\}\} \tag{10}$$

with the initial consistency condition

$$y(t_0) = \max\{x(t_0) - r, \min\{x(t_0) + r, y_0\}\} \tag{11}$$

for the output signal at initial time t_0 for piecewise monotonous input signals with a monotonicity partition $t_0 \leq t_1 \leq . . \leq t_i \leq t \leq t_{i+1} . . \leq t_N = t_E$ [9]. The play operator depends on the independent initial value $y_0 \in \Re$ of the output and is characterized by its threshold parameter $r \in \Re^+_0$. Fig. 6

shows the hysteretic region Ω which is a strip line in $\Re^2$ and the rate-independent output-input trajectory of this elementary hysteresis operator.

Although the three Madelung rules hold for the play operator it can be easily realized that the ferromagnetic, ferroelectric or elastic-plastic behaviour of real materials and the hysteretic actuator and sensor characteristics of real smart materials are of much higher complexity, note also rule 4.

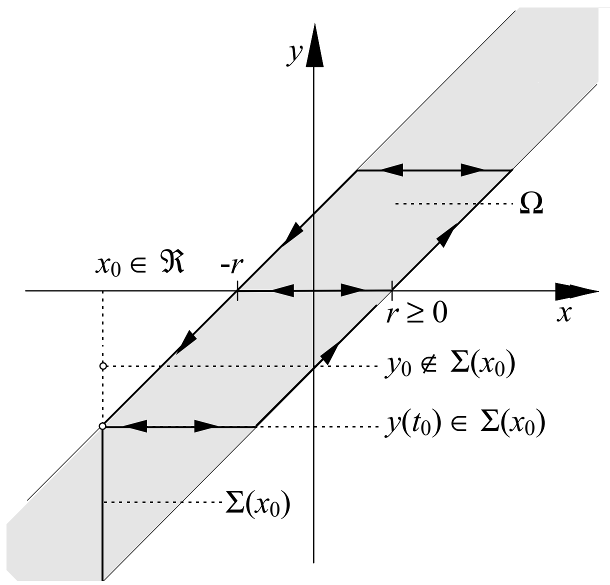

figure 6. Rate-independent characteristic of the play operator H_r

To obtain a more powerful model for complex hysteretic nonlinearities we introduce the so-called Prandtl-Ishlinskii hysteresis operator H by the linear weighted superposition of many play operators with different threshold values. From this follows

$$H[x](t) := \boldsymbol{w}^{T} \cdot \boldsymbol{H}_r[x, \boldsymbol{z}_0](t) \tag{12}$$

with the vector of weights $\boldsymbol{w}^T = (w_0\ w_1\ ..\ w_n)$, the vector of thresholds $\boldsymbol{r}^T = (r_0\ r_1\ ..\ r_n)$ with $0 = r_0 < r_1 < .. < r_n < +\infty$, the vector of the initial states $\boldsymbol{z}_0{}^T = (z_{00}\ z_{01}\ ..\ z_{0n})$ of the play operators and the vector of the play operators

$$\boldsymbol{H_r}[x, \boldsymbol{z}_0](t)^T = (H_{r_0}[x, z_{00}](t) \; . \; . H_{r_n}[x, z_{0n}](t)) \, .$$

The hysteretic characteristic of the Prandtl-Ishlinskii hysteresis operator is completely defined by the characteristic of the so-called initial loading curve. This special branch will be traversed if the initial state of the Prandtl-Ishlinskii hysteresis operator is zero and it is driven with a monotonous increasing input signal. The initial loading curve can be fully characterized by and therefore equated with a threshold-dependent piecewise linear function

$$\varphi(r) = \sum_{j=0}^{i} w_j (r - r_j) \; ; \; r_i \le r < r_{i+1} \; ; \; i = 0 \; . \; . \; n \, , \tag{13}$$

with $r_{n+1} = \infty$ and

$$\frac{\mathrm{d}}{\mathrm{d}r} \varphi(r) = \sum_{j=0}^{i} w_j \; ; \; r_i \le r < r_{i+1} \; ; \; i = 0 \; . \; . \; n \, . \tag{14}$$

It is called the generator function of the Prandtl-Ishlinskii hysteresis operator [12], see fig. 7 for a Prandtl-Ishlinskii hysteresis operator with a model order of $n = 4$.

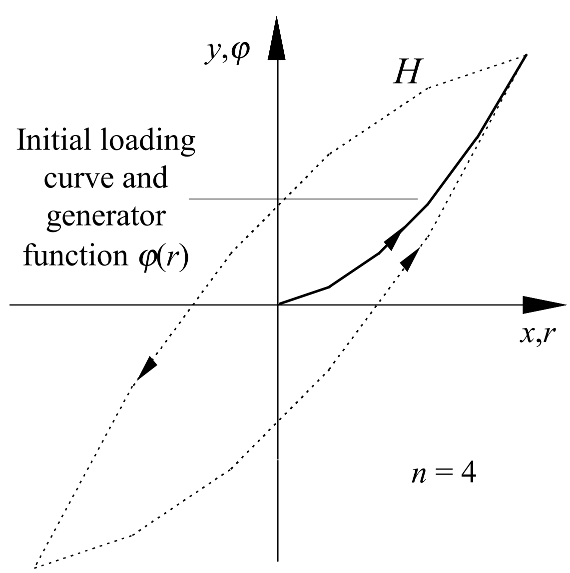

figure 7. Initial loading curve and generator function $\varphi(r)$

Under the consideration of the linear inequalitiy constraints

$$\boldsymbol{U} \cdot \boldsymbol{w} - \boldsymbol{u} \geq \boldsymbol{0} \tag{15}$$

for the weights with the matrix

$$\boldsymbol{U} = \begin{pmatrix} 1 & 0 & . & . & 0 \\ 1 & 1 & . & . & 0 \\ . & . & . & & . \\ . & . & & . & . \\ 1 & 1 & . & . & 1 \end{pmatrix}, \text{ the vector } \boldsymbol{u} = \begin{pmatrix} \varepsilon \\ \varepsilon \\ . \\ . \\ \varepsilon \end{pmatrix} \text{ and a}$$

possibly infinite small number $\varepsilon > 0$ the generator function is strongly monotonous for $r \geq 0$ and therefore the inverse of the generator function $\varphi^{-1}(r)$ exists uniquely for $r \geq 0$. $\varphi^{-1}(r)$ is piecewise linear and strongly monotonous and can therefore also be regarded as a generator function

$$\varphi'(r') = \sum_{j=0}^{i} w'_j (r' - r'_j) \;;\; r'_i \leq r' < r'_{i+1} \;;\; i = 0\,.\,.\,n\,, \tag{16}$$

of a Prandtl-Ishlinskii hysteresis operator with $r_{n+1}' = \infty$ and

$$\frac{\mathrm{d}}{\mathrm{d}r'}\varphi(r') = \sum_{j=0}^{i} w'_j \;;\; r'_i \leq r' < r'_{i+1} \;;\; i = 0\,.\,.\,n\,, \tag{17}$$

namely the inverse Prandtl-Ishlinskii hysteresis operator

$$H^{-1}[y](t) := \boldsymbol{w}'^{T} \cdot \boldsymbol{H}_{r'}[y, \boldsymbol{z}'_0](t) \tag{18}$$

with transformed initial states $\boldsymbol{z}_0'$, threshold values $\boldsymbol{r}'$ and weights $\boldsymbol{w}'$. In this case the weights fulfil the same linear inequaltity constraints

$$\boldsymbol{U} \cdot \boldsymbol{w}' - \boldsymbol{u} \leq \boldsymbol{0}\,. \tag{19}$$

The transformation law $\boldsymbol{r}' = \boldsymbol{\Omega}\,(\boldsymbol{r}\,,\boldsymbol{w})$ for the thresholds results from the relation $r'_i = \varphi(r_i)$. From this follows

$$r_i' = \sum_{j=0}^{i} w_j (r_i - r_j) \quad ; \quad i = 0\,.\,.\,n \tag{20}$$

for the threshold-discrete case, see fig. 8.

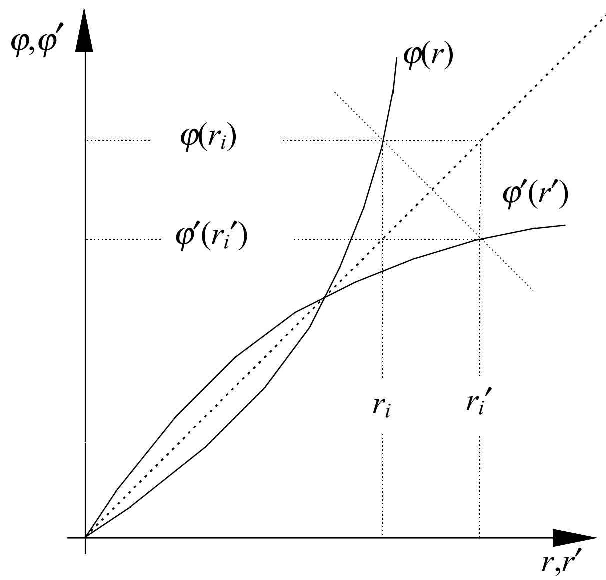

figure 8. Generator functions $\varphi(r)$ and $\varphi'(r')$

The transformation law $\boldsymbol{w}' = \boldsymbol{\Xi}(\boldsymbol{w})$ for the weights results from the relation $\mathrm{d}\varphi(r_i')/\mathrm{d}r' = 1/(\mathrm{d}\varphi(r_i)/\mathrm{d}r)$, see fig. 8. From this follows

$$w_0' = \frac{1}{w_0} \qquad \text{and} \qquad w_i' = -\frac{w_i}{(w_0 + \sum_{j=1}^{i} w_j)(w_0 + \sum_{j=1}^{i-1} w_j)} \quad ; \quad i = 1\,.\,.\,n\,. \tag{21}$$

The transformation law $\boldsymbol{z}_0' = \boldsymbol{\Psi}(\boldsymbol{z}_0,\boldsymbol{w})$ for the initial states results from the relation

$$(z_{0i+1}' - z_{0i}')/(r_{i+1}' - r_i') = (z_{0i+1} - z_{0i})/(r_{i+1} - r_i)$$

which is the threshold-discrete counterpart to the relation $\mathrm{d}z(r')/\mathrm{d}r' = \mathrm{d}z(r)/\mathrm{d}r$ for the threshold-continuous case discussed in [12]. From this follows the transformation law

$$z_{0i}' = \sum_{j=0}^{i} w_j z_{0i} + \sum_{j=i+1}^{n} w_j z_{0j} \quad ; \quad i = 0\,.\,.\,n \tag{22}$$

for the initial states.

The Prandtl-Ishlinskii hysteresis operator has the following more or less obvious properties:

1. Because the Madelung rules persist under linear superposition, they hold also for the Prandtl-Ishlinskii hysteresis operator. Moreover due to the $n > 1$ inner hysteretic state variables different branches can be traversed from a non-turning point D which is in agreement with rule 4.
2. The closed loops which will be traversed for input signals oscillating between maximum and minimum values have an even symmetry to the center point of the corresponding loop. This even symmetry property is a property of the play operator and persists also under linear superposition.
3. The inversion operation which is given by the transformation laws does not change the structure of the Prandtl-Ishlinskii hysteresis operator and its inequality constraints for the weights.

Property 1 agrees at least qualitatively with experimental observations for complex hysteretic nonlinearities. Property 3 leads to a direct formulation and thus to a very efficient implementation of the corresponding compensator which is feasible for real-time control applications. The amount of calculation for the Prandtl-Ishlinskii operator H and its compensator H^{-1} in dependence of the model order n is shown in table 1.

Table 1

Amount of calculation for the Prandtl-Ishlinskii operator H and its compensator H^{-1}

operator	addition	multiplication	comparison
H	$2(n+1)+n$	$n + 1$	$2(n+1)$
H^{-1}	$2(n+1)+n$	$n + 1$	$2(n+1)$

The even symmetry property 2 which is an inherent model characteristic is the main drawback of this Prandtl-Ishlinskii modeling approach in comparison with the Preisach hysteresis modeling approach. But in many practical cases this property is often fulfilled. Well-known examples are piezoelectric and magnetostrictive actuators driven in operating regimes with moderate input amplitudes.

The identification procedure which is used to obtain the compensator of the real hysteretic nonlinearity is divided into three parts. In the first part the thresholds $\boldsymbol{r}$ and the initial states $\boldsymbol{z}_0$ of the Prandtl-Ishlinskii hysteresis operator are determined by the formulas

$$r_i = \frac{i}{n+1} \max_{t_0 \le t \le t_e} \{|x(t)|\} \quad ; \quad i = 0 \, .. \, n \tag{23}$$

and

$$z_{0i} = 0 \quad ; \quad i = 0 \, .. \, n \, . \tag{24}$$

(24) assumes the start of the hysteretic state evolution from the so-called demagnetized state. The identification of the weights $\boldsymbol{w}$ of the Prandtl-Ishlinskii hysteresis operator which is the object of the second part can be formulated as an $L_2{}^2$-norm minimization of the so-called output error model

$$E[x, y](\boldsymbol{w}, t) := \boldsymbol{w}^T \cdot \boldsymbol{H}_r[x, \boldsymbol{z}_0](t) - y(t) \tag{25}$$

which is linearly dependent on the weights. This leads to the quadratic optimization problem

$$\min_{\boldsymbol{w} \in \Re^{n+1}} \{\boldsymbol{w}^T \cdot \int_{t_0}^{t_e} \boldsymbol{H}_r[x, \boldsymbol{z}_0](t) \boldsymbol{H}_r[x, \boldsymbol{z}_0]](t)^T \mathrm{d}t \cdot \boldsymbol{w} - \int_{t_0}^{t_e} 2 y(t) \boldsymbol{H}_r[x, \boldsymbol{z}_0]](t)^T \mathrm{d}t \cdot \boldsymbol{w} + \int_{t_0}^{t_e} y(t)^2 \mathrm{d}t\} \tag{26}$$

with the linear inequality constraints (15)

$$\boldsymbol{U} \cdot \boldsymbol{w} - \boldsymbol{u} \ge \boldsymbol{0}$$

which has one global solution and which ensures the invertability of the identified Prandtl-Ishlinskii hysteresis operator. This guarantees a unique best L_2^2-norm approximation of the measured hysteretic characteristic in that space of the weights which leads to an invertible Prandtl-Ishlinskii hysteresis operator.

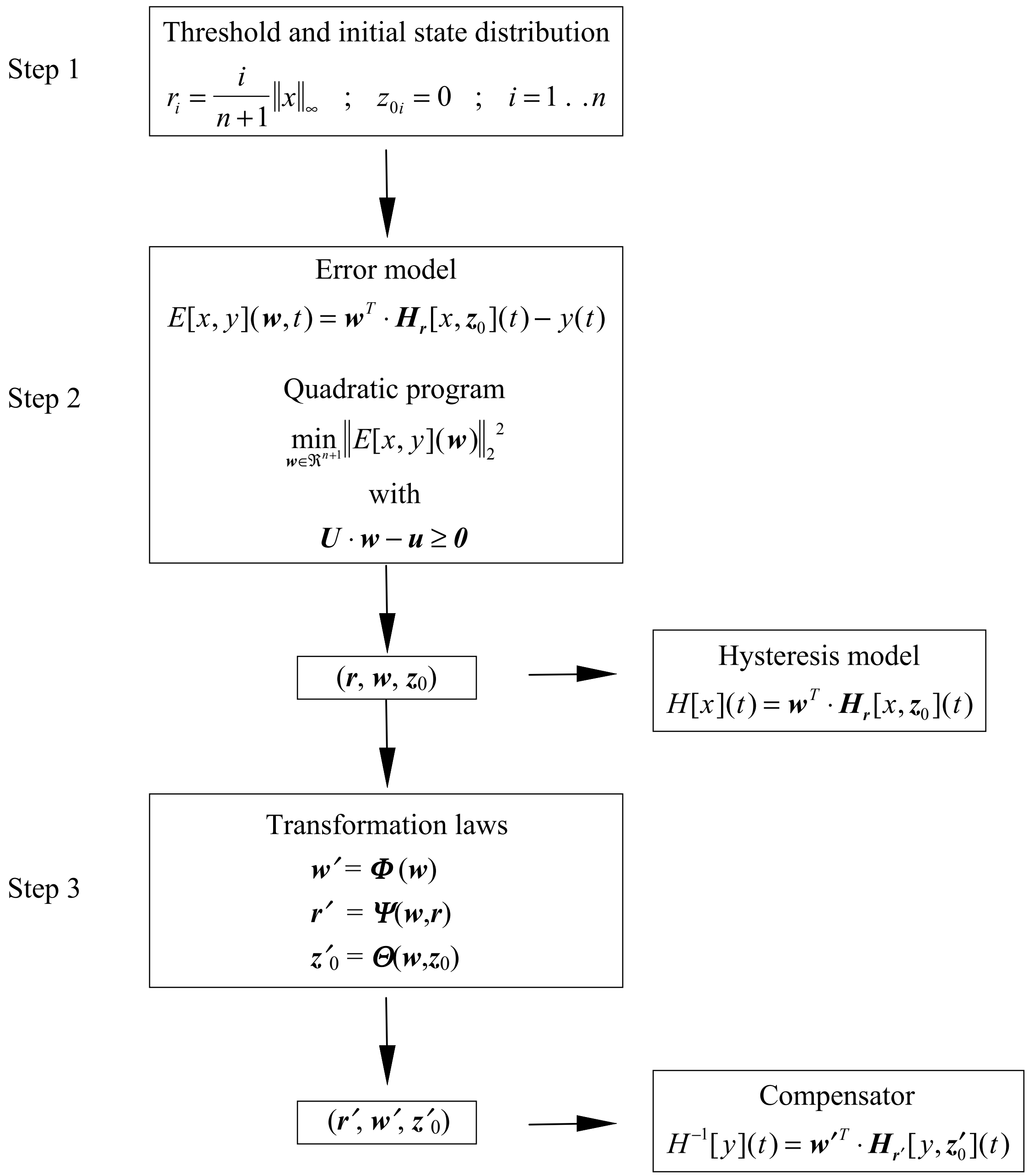

figure 9. Compensator design procedure

Therefore the invertability of the Prandtl-Ishlinskii hysteresis operator and its inverse is always guaranteed during the optimization and thus the design process for the model and the corresponding compensator is consistent and robust against unknown measurement errors of input-output data, unknown model errors and unknown model orders. The parameters of the compensator result in the third step by applying the transformation equations (20-22) to obtain the threshold, weights and initial states of the inverse Prandtl-Ishlinskii operator. Fig. 9 shows the whole compensator design procedure based on the Prandtl-Ishlinskii approach.

4 Results

In this section the performance of the presented compensator design method for complex hysteretic nonlinearities will now be demonstrated by means of the displacement-current relation of a magnetostrictive transducer, see fig. 10.

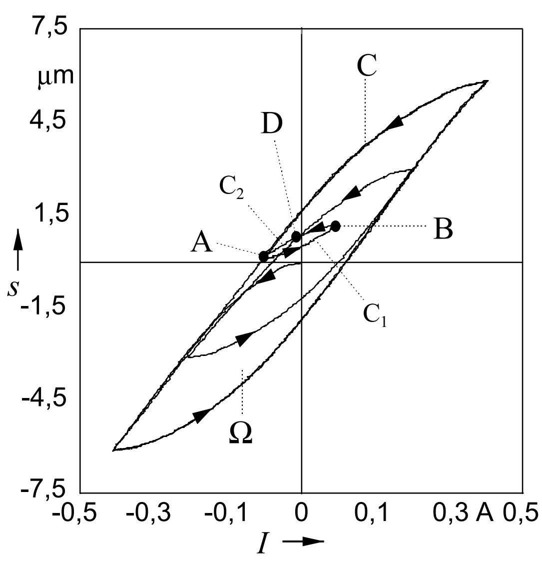

figure 10. Measured complex hysteretic displacement-current relation in the operating range

To get a strongly monotonous relation between the displacement and current which is fundamental for actuator operation, the magnetostrictive transducer is used with an additional bias current. The bias current which amounts to 1 A in this example determines the operating point of the magnetostrictive actuator. This operating point coincides with the origin of the *s*-*I*-plane in fig. 10 which shows the strongly monotonous hysteretic displacement-current relation in the moderate signal operating range of the magnetostrictive actuator. It is mainly characterized by strongly monotonous branches and nearly symmetrical hysteretic loops with a counterclockwise orientation. Additionally the real hysteretic characteristic fulfils the Madelung rules and exhibits the crossing loop property. Therefore the modeling, identification and compensation of this real complex hysteretic nonlinearity can be realized with the Prandtl-Ishlinskii approach.

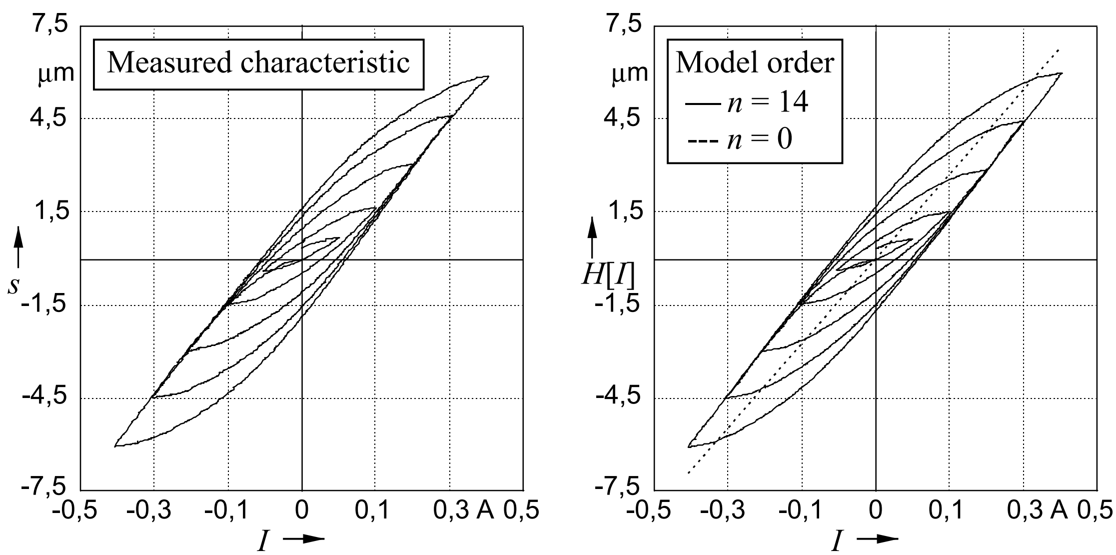

figure 11. Measured and modeled hysteretic displacement-current relation

The model order $n = 0$ leads to a linear rate-independent operator model and thus the identification procedure determines the best linear L_2^2-norm approximation of the real hysteretic nonlinearity, see fig. 11. The nonlinearity error defined by

$$\frac{\max\limits_{t_0 \le t \le t_e} \{ | H[I](t) - s(t) | \}}{\max\limits_{t_0 \le t \le t_e} \{ | H[I](t) | \}} \tag{27}$$

amounts in this case up to 29.8 %. Fig. 11 shows the looping and branching behaviour of the real complex hysteretic characteristic of the magnetostrictive actuator and the Prandtl-Ishlinskii hysteresis operator with a model order of $n = 14$ as a result of the identification procedure. The nonlinearity error amounts in this case to 3.0 % which is nearly ten times smaller as for the best linear L_2^2-norm approximation. Due to unknown model errors a further increasing of the model order doesn't improve the nonlinearity error in this case.

For the compensation of the real hysteretic nonlinearity a inverse feedforward controller is used which is based on the inverse Prandtl-Ishlinskii hysteresis operator, see fig. 12. $s_c(t)$ is the given displacement signal value.

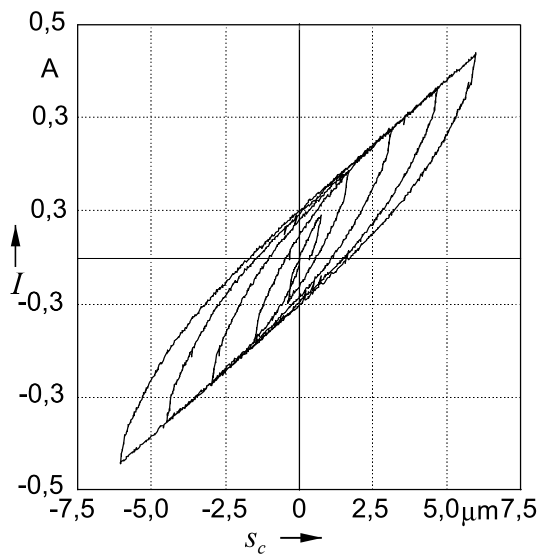

figure 12. Inverted hysteretic displacement-current relation

The inverse Prandtl-Ishlinskii hysteresis operator is obtained from the Prandtl-Ishlinskii hysteresis operator using the transformation laws for the thresholds (20), the weights (21) and the initial states (22). It is realized by a digital signal processor with a sampling rate of up to 10 kHz and a displacement controlled current source. The looping and branching characteristic of the inverse Prandtl-Ishlinskii hysteresis operator is shown in fig. 12.

As a final result fig. 13 shows the compensated characteristic of the overall system given by the serial combination of the inverse feedforward controller and the magnetostrictive actuator.

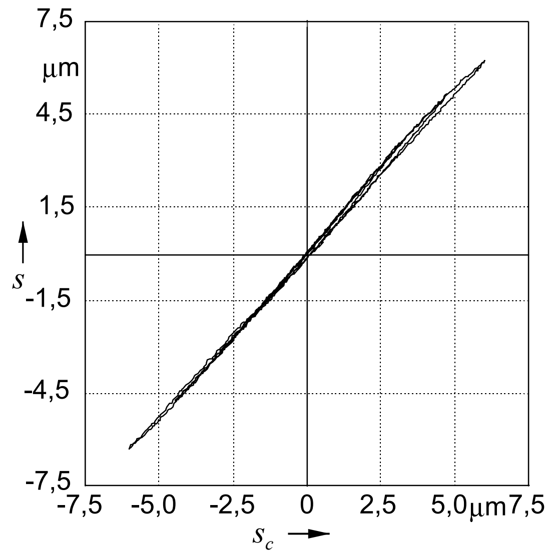

figure 13. Compensated hysteretic displacement-given displacement relation

In this example the control error defined by

$$\frac{\max\limits_{t_0 \le t \le t_e}\{|s_c(t) - s(t)|\}}{\max\limits_{t_0 \le t \le t_e}\{|s_c(t)|\}} \tag{28}$$

will be strongly reduced to about 3 % due to the inverse feedforward control strategy.

5 Conclusions

The main contribution of this paper is to extend the Prandtl-Ishlinskii modeling approach for complex hysteretic nonlinearities to a robust compensator design method for invertible complex hysteretic nonlinearities of the Prandtl-Ishlinskii type. For this purpose the threshold-discrete version of the Prandtl-Ishlinskii hysteresis operator was formulated with linear inequality constraints for the model parameters which guarantee the invertability of the model. Based on these linear inequality constraints and an error model which is linearly dependent on the model parameters the identification problem can be formulated as a quadratic program which provides always the best invertible L_2^2-norm approximation of the measured output-input data. The corresponding compensator can be directly calculated and thus efficiently implemented from the model by analytical transformation laws. Finally the compensator design method is used to generate an inverse feedforward controller for a magnetostrictive actuator. In comparison to the conventionally controlled magnetostrictive actuator the nonlinearity error of the inversely controlled magnetostrictive actuator is lowered from about 30 % to about 3 %. In future works the method will be extended to hysteresis operators which are also able to model complex hysteretic nonlinearities with asymmetrical hysteretic loops. These type of nonlinearities occur if magnetostrictive or piezoelectric actuators are driven with higher input amplitudes.

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