# COMPENSATION OF THE CREEP AND HYSTERESIS EFFECTS OF PIEZOELECTRIC ACTUATORS WITH INVERSE SYSTEMS

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Abstract:
The present paper will describe an approach for the compensation of the hysteretic and creeping transfer characteristics of a piezoelectric stack transducer by interposing an inverse system in an open loop control. The basis of the inverse control is formed by complex creep and hysteresis operators, which represent, adequately connected, a precise model for the creep and hysteretic transfer behaviour. The complex creep and hysteresis operators consist of the weighted superposition of elementary operators which, in terms of mathematics, can easily be described and which reflect the qualitative properties of the transfer characteristic of the transducer. This operator-based transducer model allows the prediction of the transfer behaviour within the inverse control in order to calculate the compensation signal. As a result the maximum linearity error caused by hysteresis and creep effects will be lowered by one order of magnitude.

**Introduction**

Piezoelectric solid-state transducers are capable of immediately transforming electric into mechanical energy or vice versa and are therefore used for industrial purposes as high dynamic actuators and as fast sensors. Especially when used as an actuator its electromechanical transfer behaviour is characterised by creep and hysteretic effects because the transducer is driving with high voltage amplitudes $x(t)$ to generate the longest possible displacements $y(t)$, see Fig. 1.

This leads to ambiguities in the transfer behaviour of piezoelectric energy transducers and thus to a considerable reduction of the repeatability attainable in an open loop control. Today in practice this disadvantage can be avoided by adjusting the position of the transducer within a closed loop control. This model, however, requires an additional displacement sensor to determine the output quantity, a controller to generate the controller output and a calibration of the displacement sensor as well as a complicated controlling adjustment. The present paper will describe an alternative solution based upon the compensation for non-ideal transfer characteristics by interposing an inverse system in an open loop control, see Fig. 2.

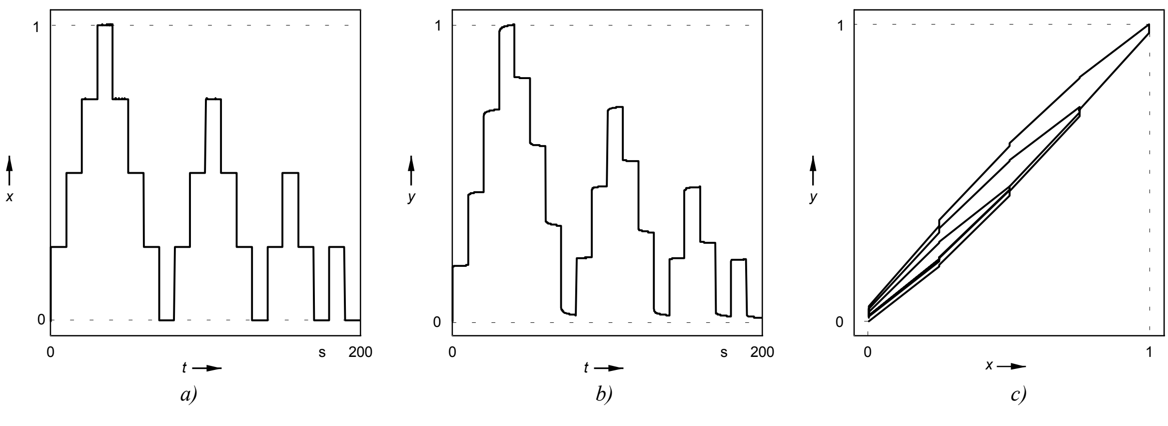



*Fig 1: Electromechanical transfer characteristic of a piezoelectric solid-state transducer as an actuator*
*a) Electrical excitation x(t), normalized on the maximum amplitude b) Mechanical reaction y(t), normalized on the maximum amplitude c) Mechanical reaction y over the electrical excitation x*

For this an operator $\Gamma_a[]$ will be developed, which describes the hysteretic and creeping large signal transfer characteristic of the transducer.

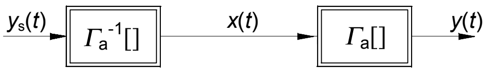



***Fig.2:*** *Signal flow chart of the hysteresis and creep free control*

Based on this operator a numerical procedure will be given, which realizes the inverse operator

$$x(t) = \Gamma_a^{-1}[y_s(t)] \; . \tag{1}$$

**Theoretical fundamentals**

In the mathematical literature the notation of the hysteretic nonlinearity will be equated with the notation "rate independent memory effect" [7]. This means that the output signal of a system with hysteresis depends not only on the present value of the input signal but also on the order of their amplitudes, especially their extremum values, but not on their rate in the past. The rate-independent branching transfer characteristic shown in Fig. 1c is a typical sign of a system with hysteretic nonlinearities. Because of its phenomenological character the concept of hysteresis operators developed by Krasnosel'skii and Pokrovskii in the 1970's allows a very general and precise modelling of hysteretic system behaviour [3]. The basic idea consists of the modelling of the real hysteretic transfer characteristic by the weighted superposition of many elementary hysteresis operators, which differ dependent on the type of the elementary operator in one or more parameters.

One type of such an elementary hysteresis operator is the so called linear play operator

$$\eta_r(t) = p_r[x(t), \eta_r(t_0), x(t_0)] . \tag{2}$$

The operator is characterized by its treshhold parameter $r$. The initial value of the operator state, namely the pair $(\eta_r(t_0), x(t_0))$, determines in a clear manner the value of the operator output $\eta_r(t)$ in dependence of the future values of the input signal $x(t)$. A procedure for the efficient numerical calculation of the operator output $\eta_r(t)$ follows from simple geometrical considerations based on the transfer characteristic shown in Fig.3 [1]. For the precise modelling of real hysteresis phenomena more linear play operators with different treshold values $r_i$ can be superimposed. This parallel connection of elementary hysteresis operators leads to the complex hysteresis operator

$$y_h(t) = H[x(t)] = \sum_{i=1}^{n} q_i \cdot p_{r_i}[x(t), \eta_{r_i}(t_0), x(t_0)] . \tag{3}$$

In practice due to the continuity of the linear play operator complex hysteresis loops can sufficiently precise modeled with the help of a small amount of elementary operators [1].

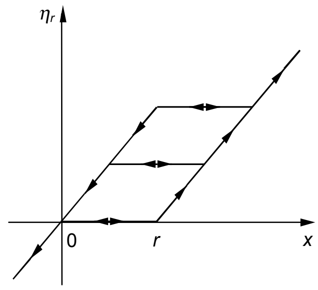



***Fig. 3:*** *Rate-independent transfer characteristic of the linear play operator*

Therefore the hysteresis operator (3) is a suitable tool for the real-time calculation of the complex hysteresis transfer characteristic.

The notation of creep originally comes from the field of solid-mechanics and describes the time-variant deformation behaviour of a body due to a sudden mechanical load [2,5]. It is a strongly damped, rate-dependent phenomenon, which can be found in a similar manner in the field of ferromagnetism and ferroelectricity. Like hysteresis phenomena the electrically induced creep effects, clearly seen in the real system reaction in Fig. 1b, have a considerable influence on the large-signal transfer characteristic of a piezoelectric transducer. If these creep phenomena are linear, they can be described, analogously to the hysteresis modelling process, by a complex linear creep operator

$$y_c(t) = L[x(t)] = \sum_{j=1}^{m} c_j \cdot l_{\lambda_j}[x(t), z_{\lambda_j}(t_0)], \tag{4}$$

which is made up by a weighted superposition of many elementary linear creep operators with different creep eigenvalues $\lambda$. In this case the elementary linear creep operator represent the solution of a linear first order differential equation with an initial value $z_\lambda(t_0)$. Fig. 4 shows the step response of the elementary linear creep operator, which has the same qualitative properties like the step response of the creep phenomena in the real system. Analogously to the complex hysteresis operator the local creep curves in Fig. 1b can sufficiently precise modeled with the help of a small amount of elementary operators [4]. So the creep operator (4) is also a suitable tool for the real-time calculation of complex linear creep effects.

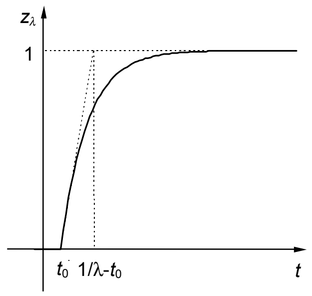



*Fig. 4: Step response of a rate dependent, elementary linear creep operator*

The real creep transfer characteristic of a piezoelectric transducer depends on the global past history of the input signal in a similar manner as the hysteretic transfer characteristic [4]. This dependence of the global past history on the input signal will not be considered by the complex linear creep operator so that it is only a crude approximation for the global creep transfer characteristic. It can be shown, that the serial connection of a complex hysteresis operator and a complex linear creep operator results in a complex nonlinear creep operator which considers the dependence of the creep effects on the global past history of the input signal in a suifficient manner [4]. The measured system reaction $y(t)$ presented in Fig. 1b shows that the electromechanical transfer characteristic of the piezoelectric transducer can be decomposed in an immediate system reaction represented by a complex hysteresis operator and a delayed system reaction represented by a complex nonlinear creep operator.

**Inverse control of the piezoelectric stack actuator**

For systems which can be described sufficiently precise by a pure strictly monotone complex hysteretis operator an inverse operator can be derived directly from the system model [6]. But here the system operator $\Gamma_a[]$ contains complex hysteresis operators and a complex linear creep operator which are coupled together and so the direct inversion of the system operator is not simply possible. Therefore the inverse system model will be carried out numerically. The assumptions for the existence of an inverse operator $\Gamma_a^{-1}[]$ and thus for the convergence of the numerical inversion procedure are the continuity and strict monotonicity of the operator $\Gamma_a[]$ [3]. Fig. 5 shows the signal flow chart of the iterative inversion procedure. In this inverse system two system models, an iteration model $\Gamma_a^{it}[]$ and a reference model $\Gamma_a^{ref}[]$, are used. Under the assumption that the real system, the iteration model and the reference model have the same state an estimated value $x_i$ for the inverse control signal $x$ is given by the algorithm block A.

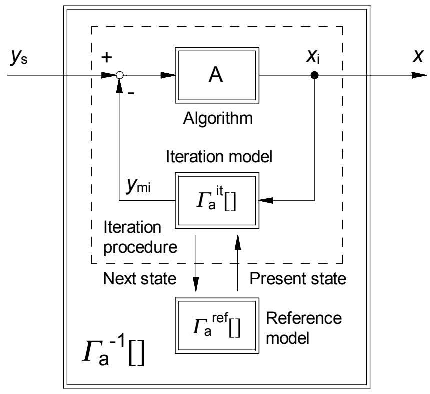



*Fig.5: Signal flow chart of the iterative inversion procedure*

With it the real system reaction $y$ due to the estimated inverse control signal value $x_i$ will be predicted by the output value $y_{mi}$ of the iteration model $\Gamma_a^{it}[]$. Then the predicted system reaction $y_{mi}$ will be compared with the given control signal value $y_s$. If the difference between $y_s$ and $y_{mi}$ is not sufficiently small, the Algorithm block A calculates an improved inverse control signal value $x_{i+1}$ based on the value $x_i$. Since the state of the iteration model $\Gamma_a^{it}[]$ has changed by the calculation of $y_{mi}$ it must be reconstructed by the reference model for the next iteration step. If the difference between $y_s$ and $y_{mi}$ is small enough the iteration procedure stops and the real system will be driven with the inverse control signal $x = x_i$. In this case the state of the real system will be the same as the state of the iteration model $\Gamma_a^{it}[]$ and thus the state of the reference model $\Gamma_a^{ref}[]$ must be updated with the state of the iteration model $\Gamma_a^{it}[]$.

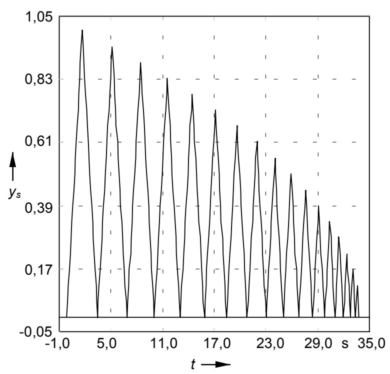
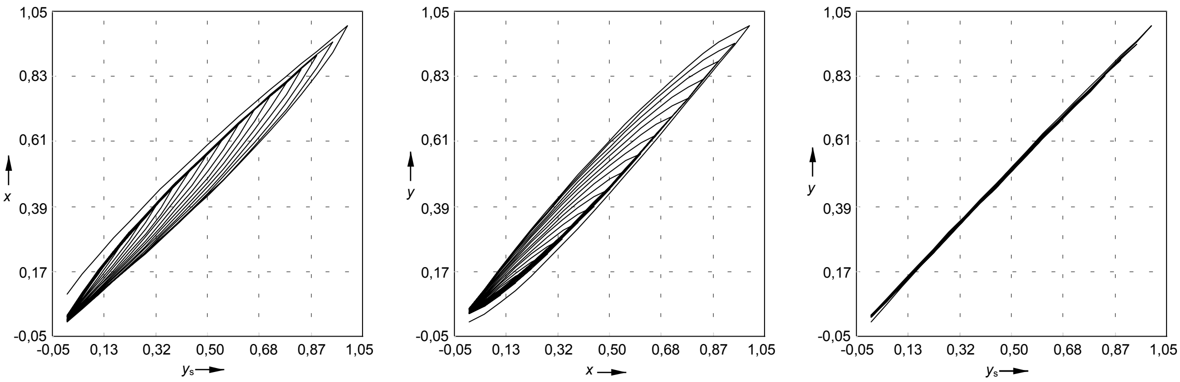



*Fig. 6: Given control signal $y_s(t)$ for the inverse system*




a) b) c)

***Fig.7:*** *Results of the inverse control process. a) Inverse control signal x versus the given control signal $y_s$. b) Output signal y versus the inverse control signal x. c) output signal y versus the given control signal $y_s$.*

**Results and Discussion**

To verify the performance of the inverse control procedure the inverse system was driven with an input signal $y_s(t)$ shown in Fig. 6. Due to the oscillating course of excitation the output-input characteristic of a high-voltage piezo stack in Fig. 7b shows strong branching effects, a typical transfer pattern for a hysteretic system. Caused by the creep effects in the transfer characteristic the hysteresis loops run through a stabilisation process at the beginning of the excitation. The relative deviation of the curve shown in Fig. 7b from the linear characteristic due to hysteresis and creep effects amouts to 11%.

Fig. 7a shows the output-input chracteristic of the inverse system. The curves run through an inverse stabilisation process due to the consideration of the creep effects by the operator-based creep model and show an inverse branching behaviour due to the consideration of hysteresis effects by the operator-based hysteresis model. Fig. 7c presents the output-input characteristic of the serial connection of the inverse system and the real system. Due to the compensation effect of the inverse control the stabilisation process and the branching behaviour caused by the creep and hysteresis phenomena are strongly reduced. In the case of the inverse control operation the relative deviation of the transfer characteristic from an optimal linear characteristic amounts only to 2%.

**Summary and prospects**

This paper has shown that complex creep and hysteresis operators offer an efficient method for the modelling and inverse control of systems with hysteretic and creep transfer characteristic. In future works this method will be extended to systems with more than one input signal. With it for example the additionally multi-valued influence of an extern mechanical load on the displacement can be considered and compensated.

**Acknowledgements**

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